

My favorite manipulatives and printed materials

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Math and Dyscalculia Services

www.DyscalculiaServices.com

www.DyscalculiaTrainingCenter.com

Common issues in KG – grade 2

- ▶ started to **count late**, difficult to count backward
- ▶ difficulty with **perception of shapes, sizes, quantities, and patterns**
- ▶ can't memorize the most basic **math facts**
- ▶ **counts on fingers** instead of using known math facts or strategies
- ▶ copies **numbers in wrong order**
- ▶ seems to **'get it'** one day, **forgets it the next**

Common issues in grades 3–5

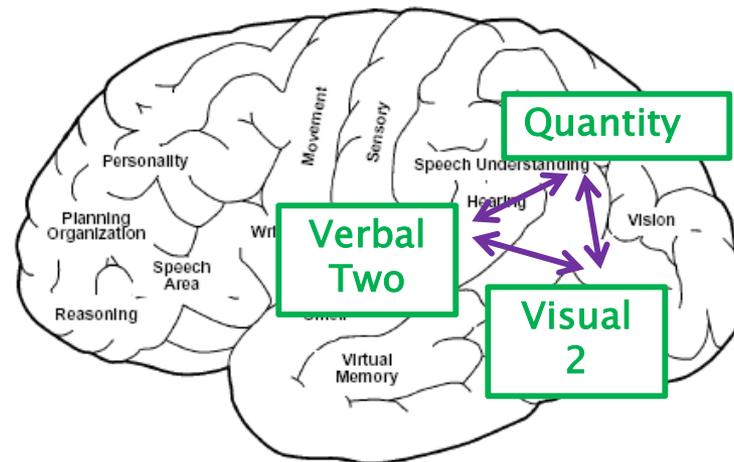
- ▶ Forgets basic facts like **multiplication tables**
- ▶ Lacks vocabulary, concepts, and procedures
- ▶ Difficulty making an **approximate guess**
- ▶ ‘Misreading’ problems: add instead of multiply
- ▶ **Word questions** are hard
- ▶ **Working slowly**, loss of confidence, avoiding Math, can mimic a behavior problem
- ▶ **Forgets numbers**: phone, dates, deadlines

make or strengthen connections

Make it *multi-sensorial* to connect the 3 math centers:

simultaneously { show the quantity **Number Sense**
say the number word(s) **Verbal**
write the numeral(s) **Visual**

1. Show the concept
2. Do it together
3. Ask student to do it
4. Repeat the concept
5. Ask student to say it



Multisensory teaching works for **all** children

- ▶ To move information from short term to long term memory and to make connections we need:

simultaneous multisensory instruction

- ▶ The sensory systems are

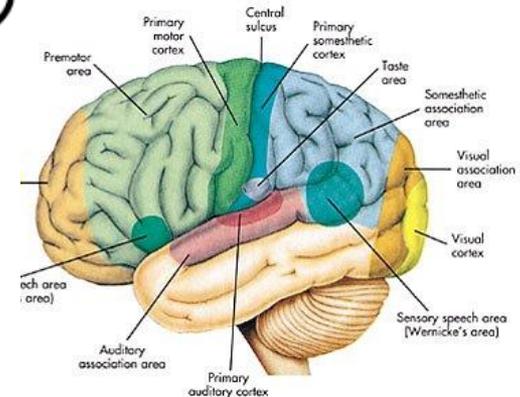
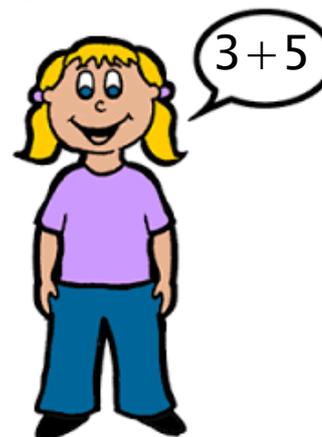
- Visual
- Auditory
- Tactile–kinesthetic
- Oral–kinesthetic

SEE

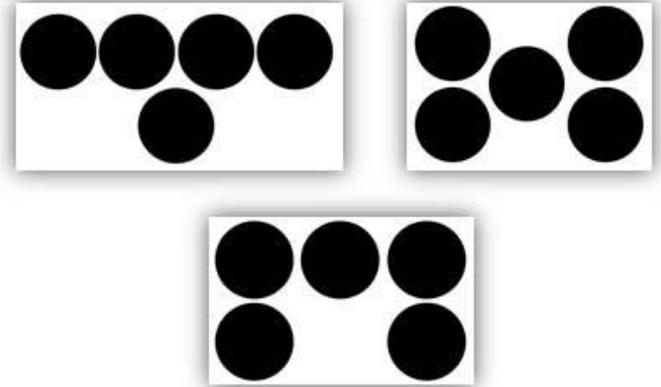
HEAR

DO

SAY



Practice subitizing



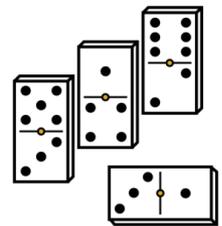
- ▶ Subitizing from Latin subitum = ‘sudden’
- ▶ Seeing the amount of objects in a set immediately **without counting**
- ▶ Most people can subitize up to five or six objects
- ▶ Dyscalculic people lack in this ability
- ▶ Subitizing is innate
- ▶ Babies can ‘count!’

Two types of subitizing

- ▶ **Perceptual**

Recognizing a number by looking at *random dot pictures* without using other mathematical processes

Perceptual subitizing helps students to separate sets and associate them with a number word to *develop counting*



- ▶ **Conceptual**

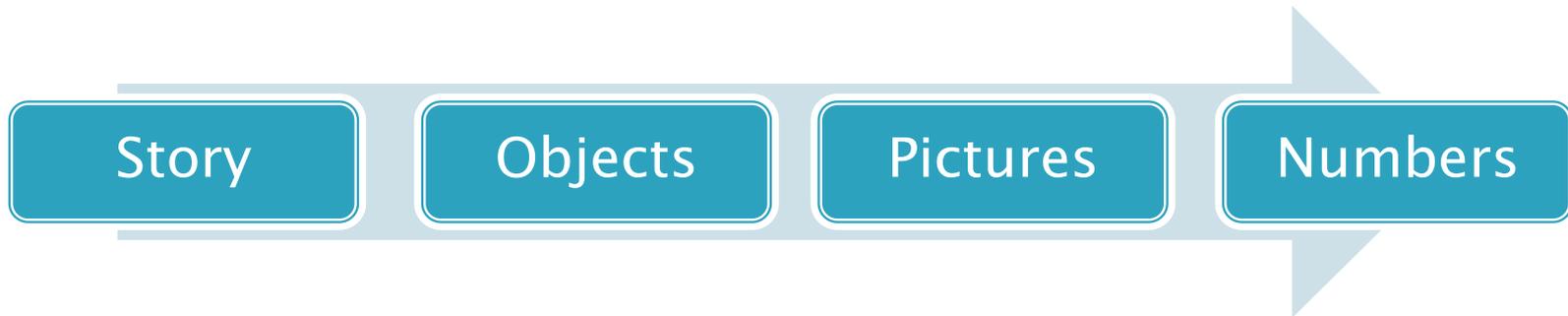
Recognizing a *familiar pattern* such as the dots on dice or dominoes

Conceptual subitizing uses patterns and helps students to *develop abstract strategies.*



How children learn math

- ▶ Children develop their math in this sequence:



- ▶ Children with dyscalculia have struggled with a step and can't make it to the next step
- ▶ We need to fill them in with **hands-on** practice

Add context to number problems:

Numbers only

- ▶ $5 + 3$
- ▶ $20 \div 5$
- ▶ **half of 12**
- ▶ $3 \times \square = 3$ quarters
- ▶ $1 \text{ ft} = ? \text{ inches}$

Story suggestions

- ▶ There are **5 coloring pencils**, you get **3 more**, how many pencils are there?
- ▶ You have **20 books** and need to pack **5 in a box**, how many boxes?
- ▶ **Half of the 12** eggs are needed for the birthday cake, how many eggs are needed?
- ▶ **3 snacks cost 3 quarters**, what does **one snack cost**?
- ▶ You have a regular **1 foot ruler**, **how many inches** does it have?

Add context to number problems:

Numbers only

▶ $5 + 3$

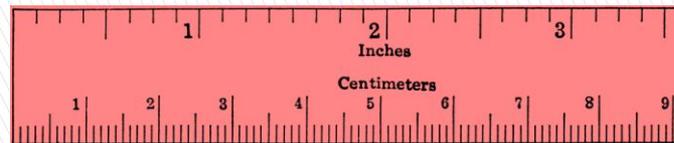
▶ $20 \div 5$

▶ half of 12

▶ $3 \times \square = 3 \text{ quarters}$

▶ $1 \text{ ft} = ? \text{ inches}$

Objects, then pictures



Just do it!



Learning Pyramid

Average Retention Rate



Source: National Training Laboratories, Bethel, Maine

Why hands on math learning?

- ▶ Learning by doing is effective and fun
- ▶ They love it, it keep's the attention going
- ▶ Use size, colors, patterns
- ▶ 'Discover' the concepts themselves
- ▶ Feeling of accomplishment
- ▶ Prevents later Math anxiety
- ▶ Building lifelong learners
- ▶ Make your own models / manipulatives:
count, color, cut, glue, fold, etc.



According to Dr Laurillard

Centre of educational Neuroscience UCL

- ▶ **Manipulations** using Cuisenaire rods, dots, patterns, dice, bead strings, counters and number lines, Numicon and Stern materials are useful to develop number sense
- ▶ **Talking** plays a key role; **the student** should describe the task, the goal, and the action **while doing it**
- ▶ **Games** help to align the student–tutor task goals and keep the attention going
- ▶ Work online with websites designated for struggling students, like **Number–Sense**

Learning Math vs. doing Math

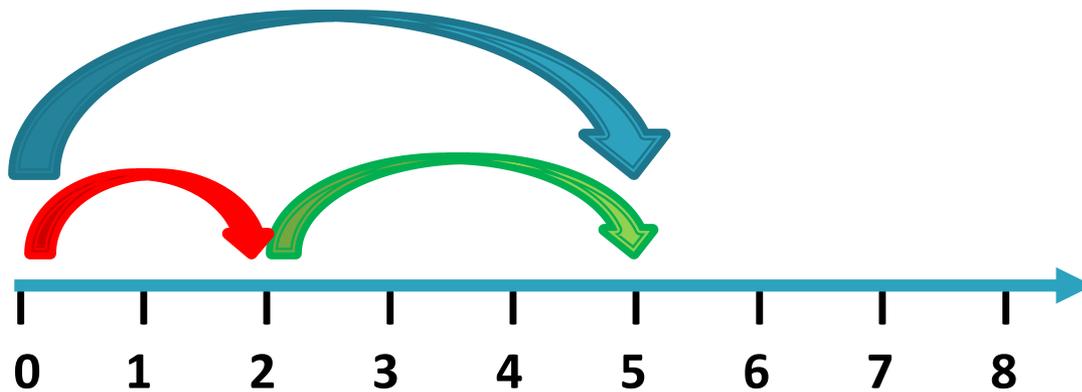
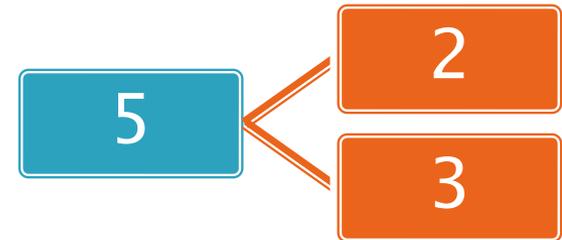
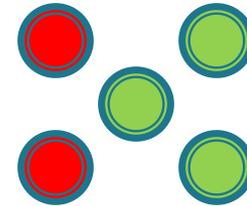
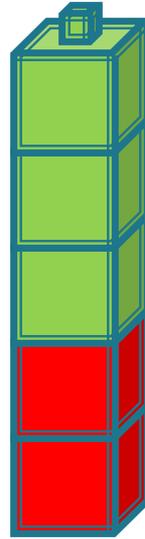
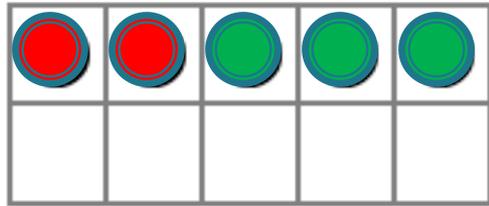
- ▶ **Almost all students can DO Math**
 - Hands on learning
 - 'Acting out' the question
 - Show one, do one together, student does one
 - Talking or writing about what they are doing



- ▶ **Almost all students love DOING Math**
- ▶ *Not all students can learn (memorize) Math*
- ▶ *Not all students love learning (memorizing) Math*



Developing Part – Part –Whole thinking



Develop fact strategies

Problem strategy

- Counts by ones
- Counts all to add

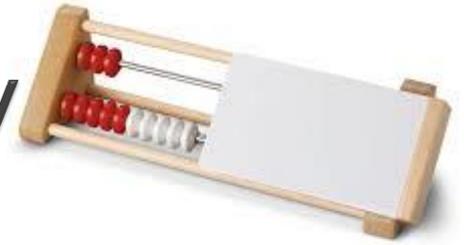
- Slow to subtract $14-4$



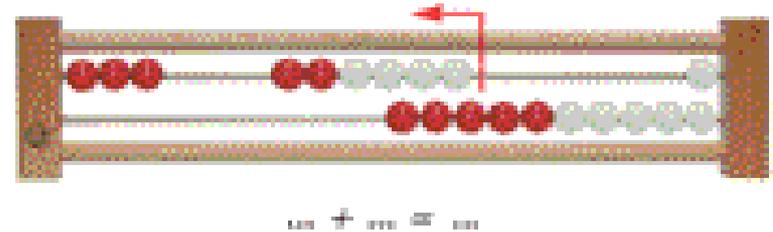
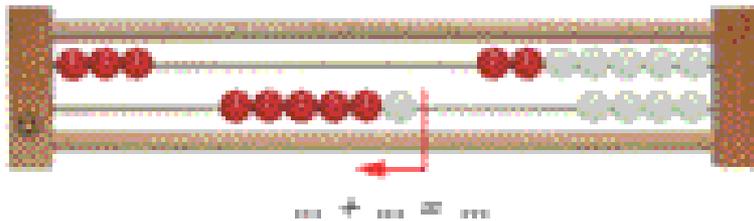
Use concrete objects to

- Model counting by 2, 5, 10
- Model counting up from the largest number, little box
- Teach ten bonds to add up to the next ten number
- Use base ten blocks and stacking number cards, take away a whole column and a card, go to 'trading' later
- Model counting back by tens
- Use number line 'jumps'

Rekenrek to develop fluency



- ▶ Fluency means flexibly working with numbers, using composing and decomposing numbers it's not just speed by rote



- ▶ Flexibility means you can choose from strategies like for $3 + 6$

Rekenrek: visualize 5s and 10s

- ▶ Subitize: Ask how did you see it?
- ▶ No counting: one push per row, think first!
- ▶ Doubles to 10: how many red, how many white?
- ▶ Add and subtract, unknown addend, unknown starting number, easy multiplication
- ▶ Strategies:
 - ❖ Make 5s or 10s
 - ❖ Doubles and near doubles
 - ❖ Make 10 first, then add / subtract some more
 - ❖ Compensate
 - ❖ Think: how many are missing on a row of 10?
 - ❖ Think: how many are missing to make 20?

Strategies: play out with counters

- ❖ Doubles, and near doubles

$$6 + 6 = 12, \text{ so } 6 + 7 = 13$$

- ❖ For $+ 9$, do $+10 - 1$

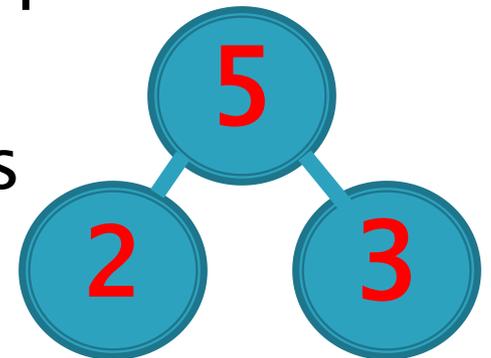
- ❖ For $- 9$, do $- 10 + 1$

- ❖ Split up numbers and do in 2 steps:

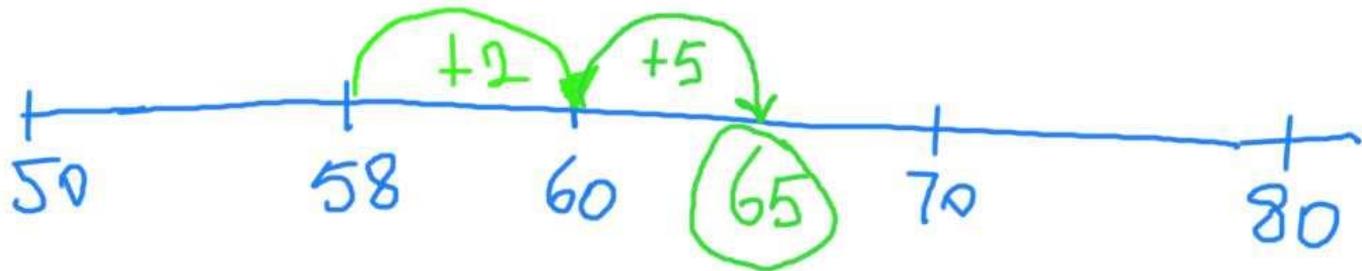
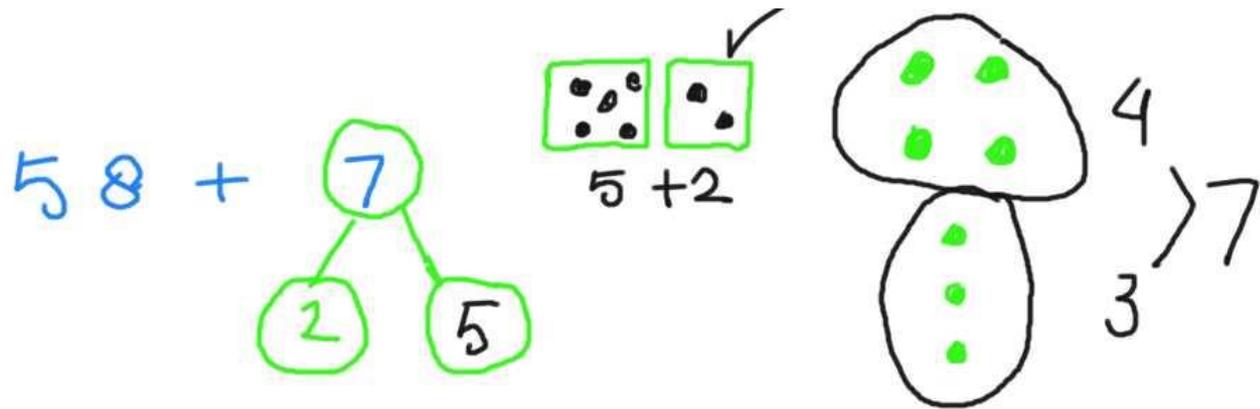
$$8 + 5$$

$$8 + 2 + 3$$

practice triads

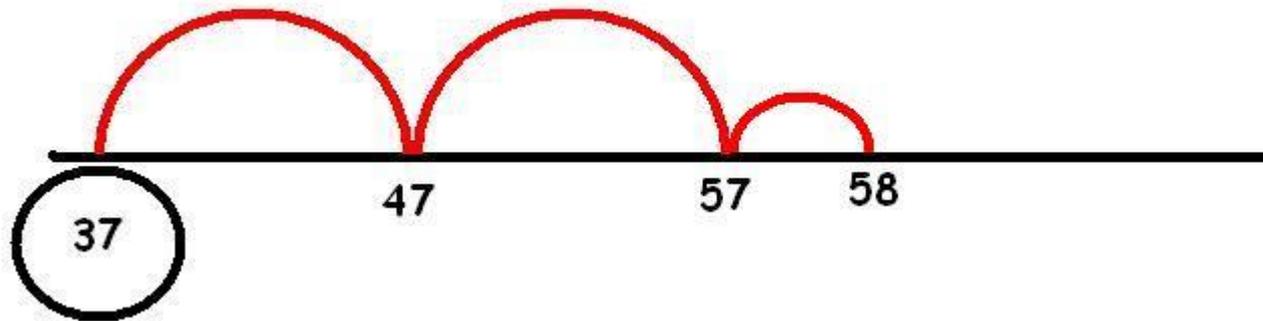


Add bridging on a 'ten' number



Add tens and ones

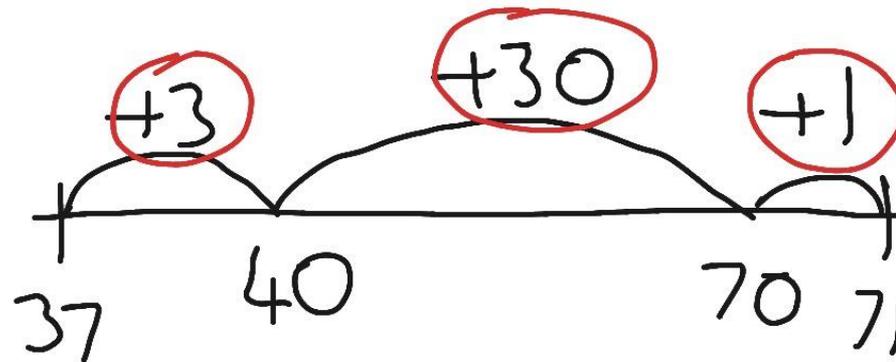
- ▶ $37 + 21 =$
- ▶ Add 2 tens and a one



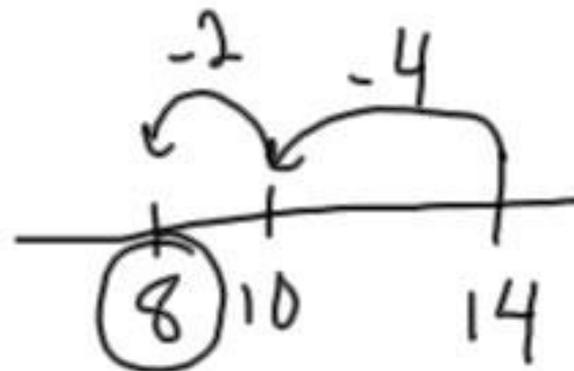
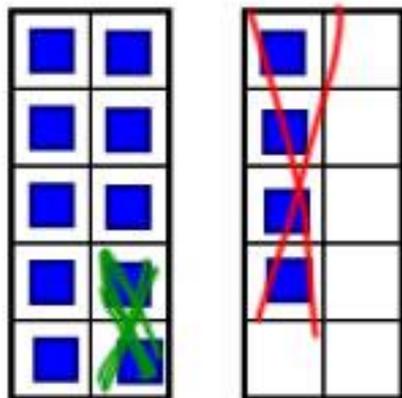
Shop keeper's method for subtraction: count up from the smallest number

.

$$71 - 37 = 34$$



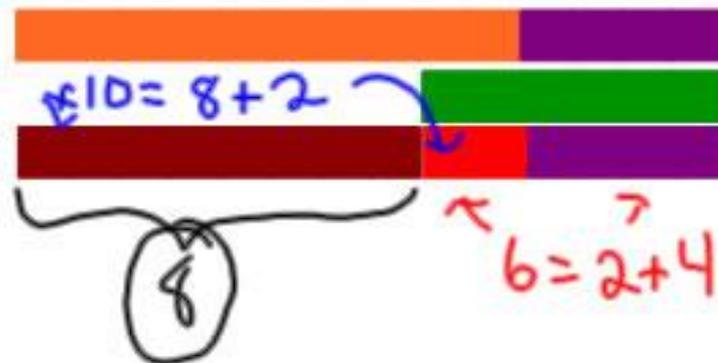
Develop strategies for $14 - 6$



$$\downarrow \quad 14 - 4 = 10$$

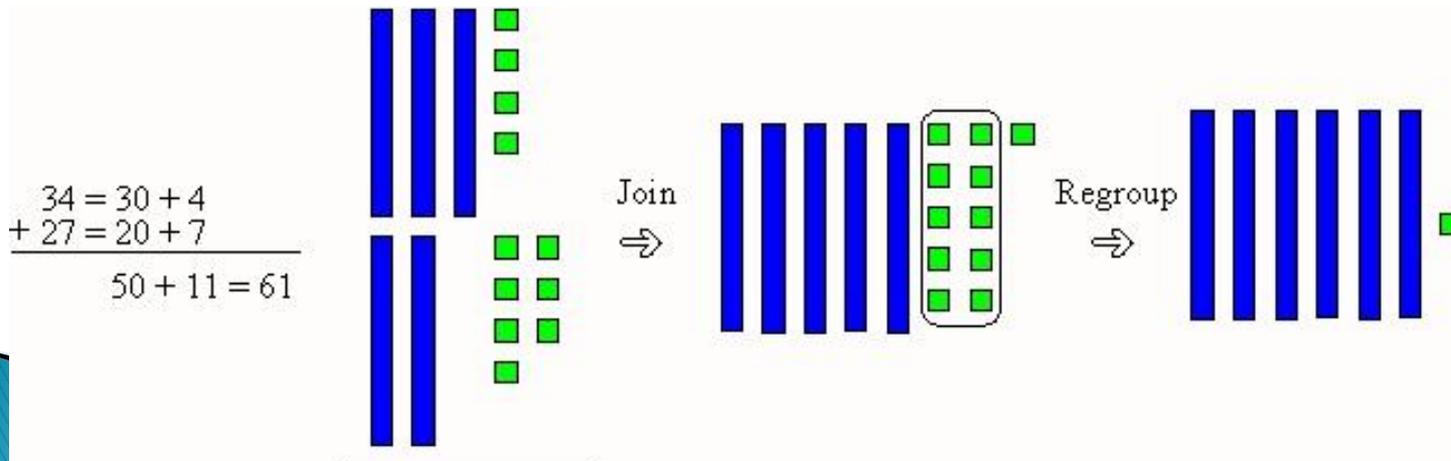
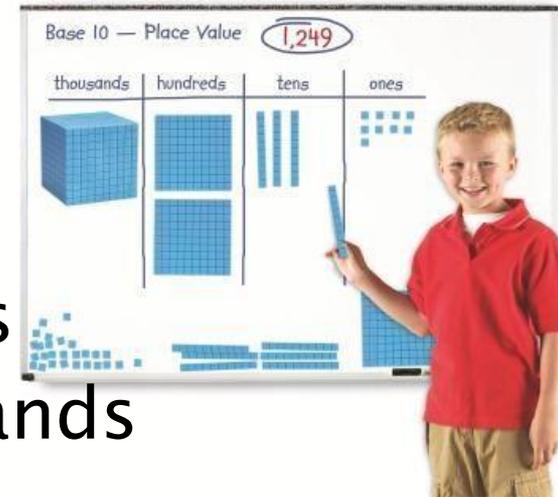
$$10 - 2 = 8$$

$$\begin{array}{r} 14 - 6 \\ \quad \swarrow \quad \searrow \\ \quad 4 \quad 2 \\ 14 - 4 = 10 \\ 10 - 2 = 8 \end{array}$$



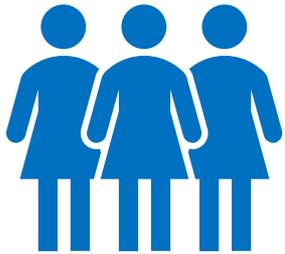
Base-10-blocks

- ▶ Visualize and count big numbers
- ▶ Count by tens, hundreds, thousands
- ▶ Place value with zeros
- ▶ Add and subtract with trading (magic ten)
- ▶ Student can show his work with concrete materials, and tell why it works, before



Developing / discovering the multiplication–division connection

- ▶ 3 kids each bring 4 balls to tennis practice

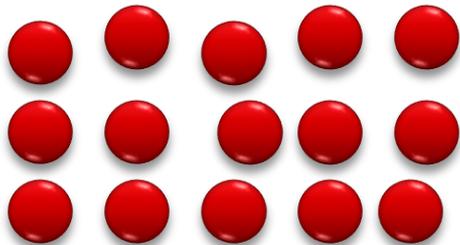


- ▶ They play with $3 \times 4 = 12$ balls
- ▶ After the practice, they divide the balls and each child takes her own $12 \div 3 = 4$ balls

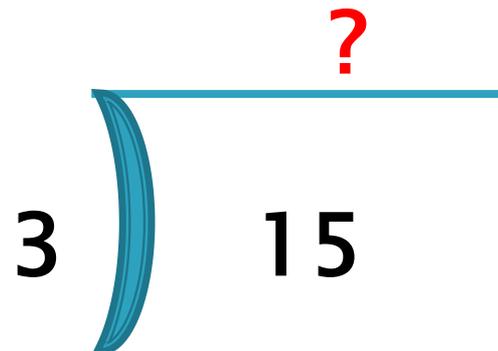
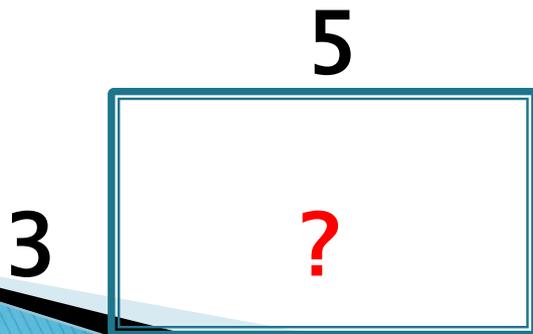


Developing / discovering the multiplication – division algorithm

- ▶ $3 \times 5 = ?$ make a row of five, make 2 more rows



- ▶ $15 \div 3 = ?$ 15 counters divided with three people
- ▶ What do you do? Hint: make 3 rows
- ▶ Or think: how many threes are there in 15?



Flash Cards & Times Tests

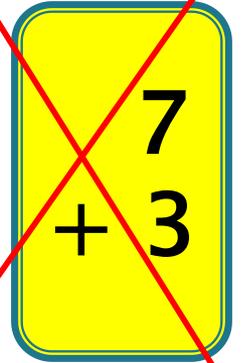


- Often used to teach rote, without understanding
- Liked by those who don't need them
- Give the false impression that math isn't about thinking
- Often produce stress – children under stress stop learning
- Not concrete – using abstract symbols
- Can lead to disliking math in general
- Result in short-term learning at best

When (not) to use flashcards

- ▶ only for review
- ▶ not for teaching new material
- ▶ often used too early in the learning process so kids can only apply rote memory

- ▶ try ‘rich’ flashcards to promote understanding
- ▶ talk about how you know the answer
- ▶ ask children to share their thinking
- ▶ give ample time

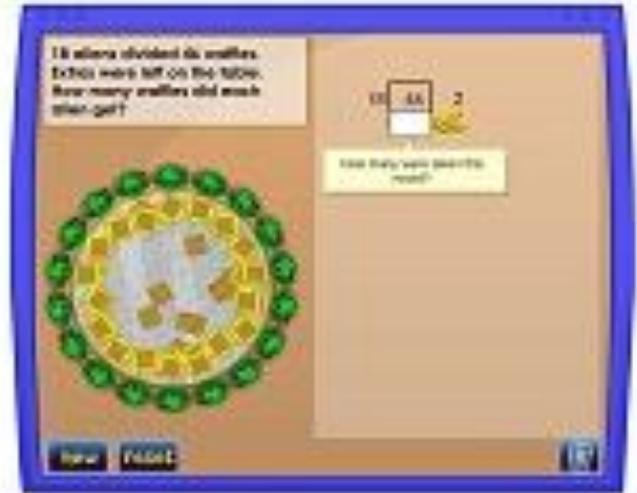


Multiplication and division practice

- ▶ Bunny times on NCTM illuminations

<https://www.youtube.com/watch?v=KIOVX-6dWYI&feature=youtu.be>

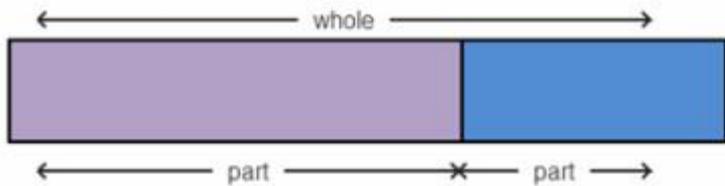
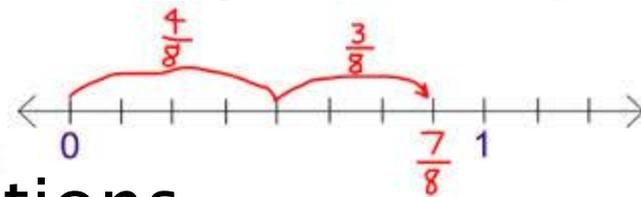
- ▶ Big 7 division in parts:
NCTM illuminations
Quotient Cafe



- ▶ Dare to Share Fairly at Math Playground

Make it 'real', use a story to 'act out' word problems

- ▶ use small objects,
- ▶ draw pictures,
- ▶ make a number-line,
- ▶ fold paper to show fractions,
- ▶ cut strips

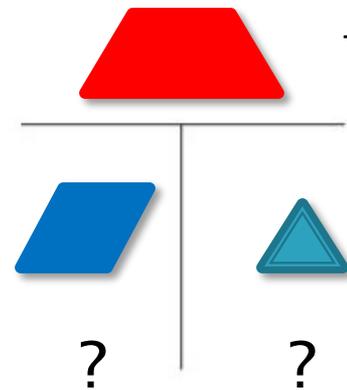
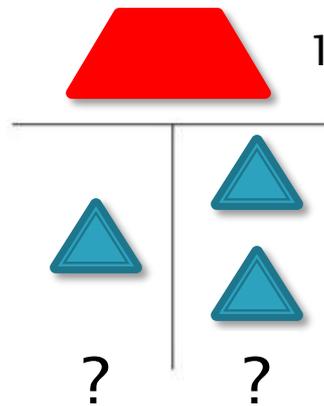
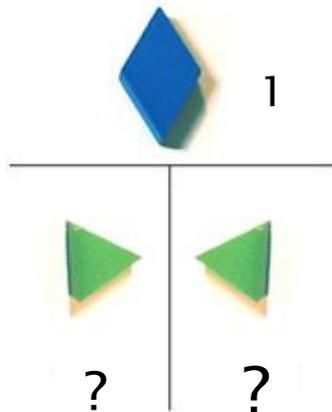
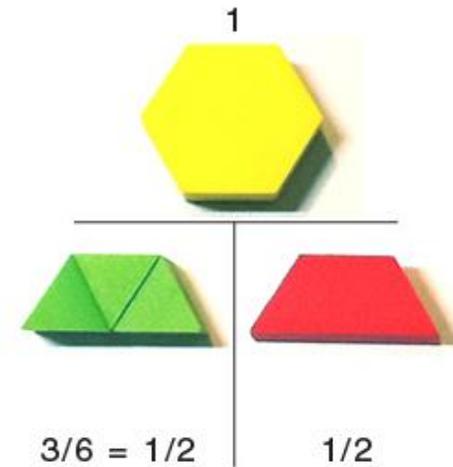
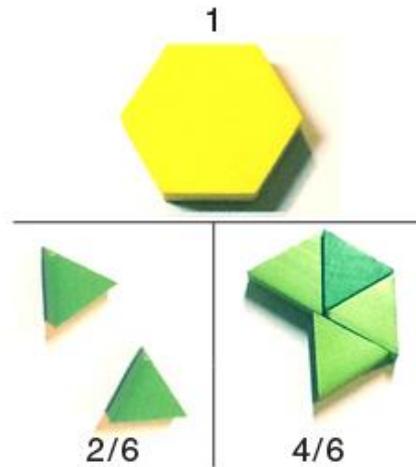
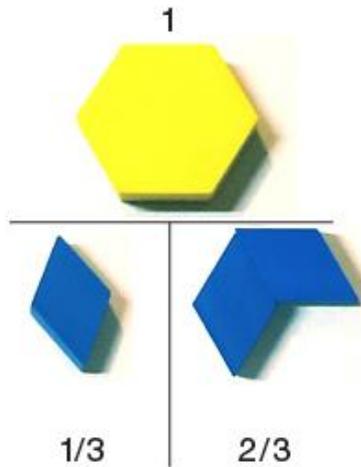


$$\text{Part} + \text{Part} = \text{Whole}$$

$$\text{Whole} - \text{Part} = \text{Part}$$



Part Part Whole in fractions



Reduce time stress



- ▶ emphasis on speed sends the wrong message: in math you should know an answer without thinking
- ▶ as result kids quickly give many answers
- ▶ emphasis on speed is related with math anxiety*
- ▶ math is not a race, but a thinking game
- ▶ speed will naturally develop with practice

* Prof Jo Boaler, Mathematics Education Stanford, 2014

Attitudes of effective math learners,

Marilyn Burns 2015



- ▶ interest to figure it out
- ▶ persevere
- ▶ check solutions in a different way
- ▶ willing to risk making errors
- ▶ accept frustrations
- ▶ not knowing an answer immediately only means **not having figured it out *yet***

Turn Math anxiety into positive feelings about math

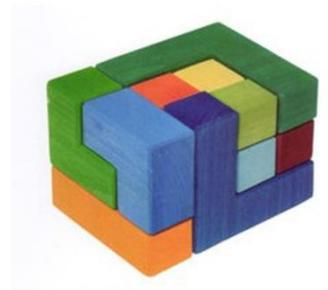
- ▶ Growth vs. fixed mindset Carol Dweck



Fixed mindset is linked to anxiety

- “I’m not going to try that difficult puzzle, what if I can’t do it, I will not be smart anymore?”

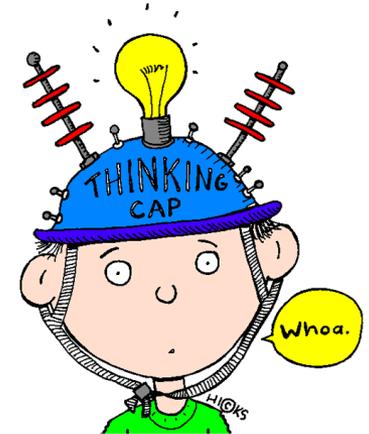
Growth mindset is linked to perseverance



- ▶ Give praise the right way:
 - ✓ Say: Super, you worked hard on this question
 - ✓ You already got a long way, great, keep trying
 - Don’t say: You did it right, you are so smart!

Mistakes grow your brain

- ▶ children learn most when something ‘seems not to be right’
- ▶ Ask How did you get there?
How do you know it’s right?
Is there an other way?
What did you estimate?
Does your answer make sense?
What if it does not make sense?
How can you mend it?



Dyscalculia: issues with connecting 3 brain centers, involved with math

- ▶ Subitizing, numerosity perception
 - ▶ Magnitude perception/processing
 - ▶ Verbal information
 - ▶ Visual information
-
- ▶ So we are looking for programs that train these capacities and provide connections between them

Common classroom accommodations

- ▶ Less stress: more time for tests and quizzes
- ▶ Make corrections mandatory
- ▶ Sit close to the teacher
- ▶ Get immediate feed back
- ▶ Use graph paper
- ▶ Allow manipulatives
- ▶ Allow graphic organizers
- ▶ Get a math buddy
- ▶ Share teacher's notes or links to online lessons

Dyscalculia curriculum options

Different curriculum

For students who cannot benefit from regular class

- ▶ Stern Math
- ▶ Numicon / Ten Frame Tower,
- ▶ Number Worlds
- ▶ Pirate Math
- ▶ **M**STAR intervention,
- ▶ Specialized tutoring
- ▶ Online learning, dyscalculia websites

Existing curriculum + extra's

- ▶ **Act out** the questions from the textbook:
connect real objects, verbal + visual input
- ▶ Graph paper
- ▶ Visual organizers
- ▶ (Connecting) counters
- ▶ Ten frames
- ▶ Number lines
- ▶ Cuisenaire rods, track
- ▶ Base-10-blocks and place value mat

Rationale of Stern Math

- ▶ Draws upon the innate ability to **recognize size**
- ▶ Children **experiment** with **sizes** and **numbers**
- ▶ **Games** are **fun**
- ▶ Material uses a **visual** and **tactile** way to show concepts that are hard to explain in words
- ▶ **Very structured**: a new concept is introduced when the necessary basis is understood
- ▶ End of year assessments also used for placement

Visual	Fraction	Decimal	Percent	
	$\frac{1}{4}$	$\frac{1}{100}$.00	%
	$\frac{4}{4}$	$\frac{4}{100}$.00	%
	$\frac{2}{2}$	$\frac{2}{100}$.00	%
	$\frac{4}{4}$	$\frac{4}{100}$.00	%
	$\frac{10}{10}$	$\frac{10}{100}$.00	%
	$\frac{100}{100}$	$\frac{100}{100}$.00	%

★ MSTAR INTERVENTION

- ▶ Meadows Center @ UT–Austin
- ▶ Aligned with Curriculum Focal Points (NCTM)
- ▶ and with Texas Essential Knowledge and Skills
- ▶ screen shot
<http://www.meadowscenter.org/institutes/mstar/mathematics-institute/mstar-intervention-facts-patterns/lessons>

Curriculum online

- ▶ **Matific**
- ▶ **Learn Zillion**: elementary and middle school
- ▶ **Khan Academy**: Math challenge with all levels has a child version, new: spiral review

Find more links with descriptions:

<http://www.edutecher.net/links.php>

Online programs for dyscalculia

- ▶ The Number Race, The Number Catcher (France) free
<http://thenumberrace.com/nr/home.php> and
<http://thenumbercatcher.com/nc/home.php>
- ▶ Meister Cody www.meistercody.com
- ▶ Calcularis, focus on number lines (Zwitserland)
<http://www.calcularis.ch/int/>
- ▶ Dynamo Maths (UK) free trial
<http://www.learning-works.org.uk/product-focus/dynamo-maths>

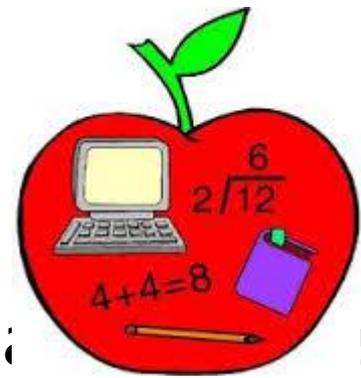
How to choose online math games

- ▶ What does my student need to learn?
- ▶ Emphasize conceptual understanding
- ▶ Balance entertainment and learning
- ▶ How 'busy' is the screen?
- ▶ How many adds?
- ▶ How are mistakes handled?
- ▶ Reward for getting things right?
- ▶ Do you or a parent see the results?

Specific info online math games

- ▶ Need a complete curriculum?
- ▶ Only need to work on a specific skill or concept ?
- ▶ Is the math content level right for this student?
- ▶ Is the math strategy right for this student?
- ▶ Is there an explanation after a mistake
- ▶ Is a new level presented when appropriate?
- ▶ Is a follow up skill presented when appropriate?
- ▶ Does the program make connections between different representations ?
- ▶ Is the presentation age appropriate?

Pc/laptop math games



- ▶ A large number of great activities : games, per grade level K–12 ‘illuminations’ from NCTM

<http://illuminations.nctm.org>

- ▶ **Thinking Blocks** uses all types of Singapore Math strip models for word problems, on iPad and PC www.mathplayground.com

Download board games



- ▶ Mixed operations games by Teresa Evans for K–5
www.makingmathmorefun.com downloads:
http://www.mathematicshed.com/uploads/1/2/5/7/12572836/making_math_more_fun_math_board_games.pdf
- ▶ <http://www.mathsphere.co.uk/resources/MathSphereFreeResourcesBoardgames.htm>
- ▶ www.activityvillage.co.uk
- ▶ www.education.com
- ▶ Middle school: Nrichmaths.org Cambridge UK



iPad Math games: keep it simple

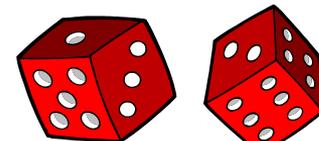
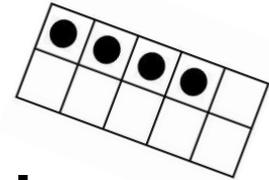
- ▶ **Make10Plus** to learn ten bonds
- ▶ **Number Rack**
- ▶ **Number Lines** frog jumps
- ▶ **Find Sums with Apples** or numbers
- ▶ **Thinking Blocks**
- ▶ **SushiMonster**
- ▶ **LobsterDiver**
- ▶ **Door 24 Math**
- ▶ **Concentration** by NCTM
- ▶ **Wings multiplication** by Mo



Effective remediation



- ▶ Individualize, focus on specific gaps
- ▶ Use concrete materials
- ▶ Show a concept in different ways
- ▶ Ask questions to get the child thinking
- ▶ Reasoning reduces need for memorization
- ▶ Start at a (very) easy level: success is important to restore self confidence
- ▶ Provide lots of practice (games)
- ▶ Make learning active and fun



When do you start?

As early as you see a problem

- Work on understanding
- Use hands on learning: **manipulatives, games**
- Provide **visual mnemonics / graphic organizers**
- **Structured, lots of repetition** (add online learning)
- Start at a (very) **easy level** to build self confidence
- **Keep track** of the progress, change the approach if needed

RTI: Keeping track in Math



- ▶ Are your students making progress?
- ▶ If you offer extra help, does the kid catch up?
- ▶ How do you know when you need the next step?
- ▶ How do you know if you need to adapt the program or look for something totally different?
- ▶ Graph the results:
 - Give extra help after two months no progress
 - Plan screening after two months no progress with help

Response to Intervention:

do they make (enough) progress?

which topics need more attention?

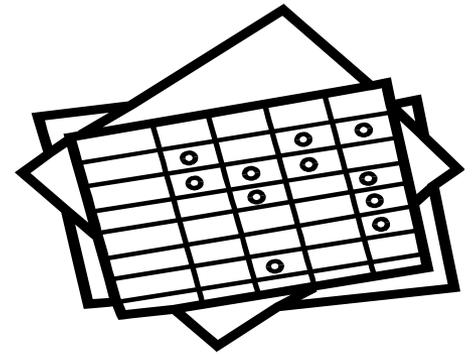
1. Focus on students' **conceptual understanding**

- **NCTMs' Focal Points Standards list**
- **TEN Test of Early Numeracy** has EN-CBM progress monitoring tools for OC, NI, QD, and MN and benchmarks 3 times a year at aimsweb.com (KG–1st grade)
- **RTI system from Oregon University**
 - monthly evaluations per grade from KG – 8th grade
 - 9 evaluations have the same difficulty (middle of the grade) to show progress over the year
 - focus is on the four basic operations, fractions, decimals, algebra, measurement, geometry, and some word problems

2. Focus on **computational skills and fluency**

informal during lessons, as tests can discourage them

EasyCBM.com demo



- ▶ Home: <http://www.easycbm.com>
- ▶ Pen and paper or online Measures
- ▶ Demo: click on ‘screenshot tour’ and scroll all the way down to NEWEasyCBMVideo (7 min.)
- ▶ After making a log in you find the Measures(Reading and Math)
<http://www.easycbm.com/teachers/auth/measures.php>
- ▶ Teachers enter their class list and click on a Measure for each kid
- ▶ Students type in their teacher’s username, select their name from a drop-down list, and choose the one they will be taking that day
- ▶ KG and first grade have a read aloud option with headphones
- ▶ Scores are presented in an excel file, interpreting the results:
<http://www.easycbm.com/static/files/pdfs/info/ProgMonScoreInterpretation.pdf>

Dyscalculia conclusion

- ▶ It runs in families
- ▶ As common as dyslexia: **around 5%**
- ▶ Not widely known, so usually not diagnosed
- ▶ Brain works differently, as seen on fMRI
- ▶ Main math centers do not (yet) work together
- ▶ Working memory issues
- ▶ Combinations with **dyslexia, ADHD, dysgraphia**
- ▶ If untreated it has **profound influence** on self esteem, schooling, and opportunities in later life
- ▶ Act upon warning signs, ***please don't delay***

What is Dyscalculia

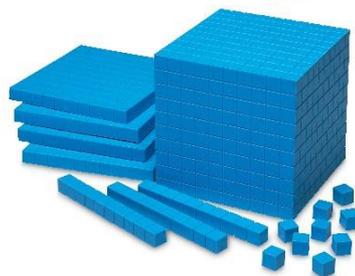
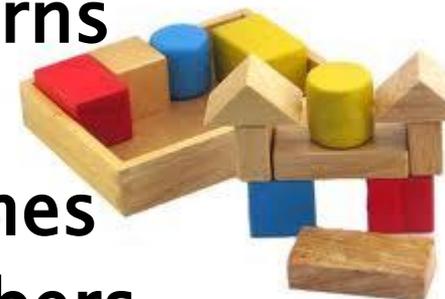


- ▶ **dyscalculia runs in families**
- ▶ **as common as dyslexia: around 4 – 7%**
- ▶ **not widely known, so usually not diagnosed**
- ▶ **brain works differently, as seen on f-MRI**
- ▶ **math centers do not (yet) work together**
- ▶ **if untreated it has profound influence on self esteem, education, and later life**
- ▶ **act upon warning signs, *please don't delay***

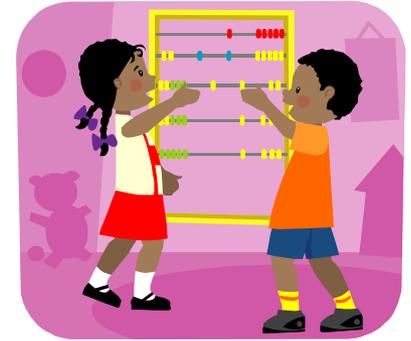
Hands-on activities and games work



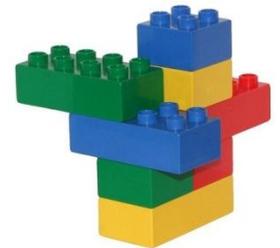
- ▶ Math activities should be fun and intuitive
- ▶ Play with shapes, sizes, colors, patterns
- ▶ Play with construction toys
- ▶ Do a lot of counting and sorting games
- ▶ Read out loud from books with numbers



Start young to promote Math with fun, hands on activities

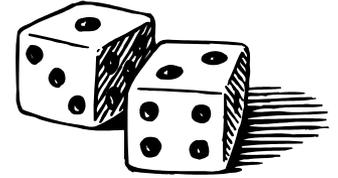


- ▶ Math activities should be fun and intuitive
- ▶ Never say something negative about math
- ▶ Play with numbers, shapes, sizes, patterns
- ▶ Provide construction toys
- ▶ Start a math journal
- ▶ Read books with numbers
- ▶ Play board games, chess
- ▶ Try math riddles

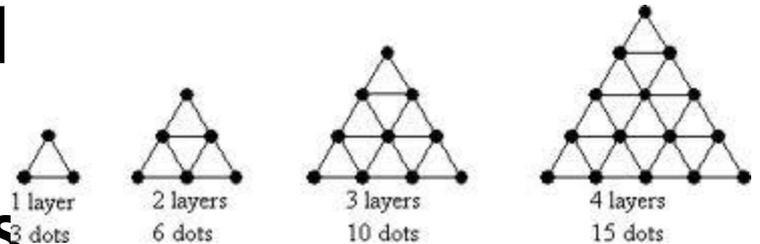


Promote Math intelligence:

goal is to connect 3 math centers:
Number sense, words, and numbers



- ▶ Math activities should be fun and intuitive
- ▶ Play with shapes, sizes, colors, patterns
- ▶ Use construction material
- ▶ Play board games, chess
- ▶ Read books with numbers
- ▶ Try math riddles and competitions
- ▶ Set up a **Math Fair** this year, enter competition at **[MindResearch.org/gameathon/](https://www.mindresearch.org/gameathon/)**



Golden rules for dyscalculia

- ▶ **Use real life examples, manipulatives and models**
- ▶ **Never say anything the student can say**
- ▶ **Immediate, constructive feedback**
- ▶ **Never drill anything that's not deeply understood**
- ▶ **Work one on one, in the student's pace**
- ▶ **Start at an (very) easy level**
- ▶ **Make repeated practice fun with games**
- ▶ **Work on self confidence, show progress**

Take home



- 1. Hands-on learning is good for all and will prevent many students from falling behind**
- 2. Screen all students for dyscalculia**
- 3. Give help as early as possible**
- 4. If no or little progress: start specialized dyscalculia tutoring**

Online courses and webinars

- ▶ Dyscalculia Awareness Course
- ▶ Moms Teach Math for homework help
- ▶ interactive Dyscalculia Online Training
www.DyscalculiaTrainingCenter.org

